Experimental and Theoretical Results in Output-Trajectory Redesign for Flexible Structures[1]

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In this paper we study the optimal redesign of output trajectories for linear invertible systems. This is particularly important for tracking control of flexible structures because the input-state trajectories, that achieve tracking of the required output, may cause excessive vibrations in the structure. Therefore, a trade-off is required between tracking and vibration reduction. We pose and solve this problem, in the context of linear systems, as the minimization of a quadratic cost function. The theory is developed and applied to the output tracking of a flexible structure and experimental results are presented.

1 Introduction

Large structures, like manipulators for assembling the space station, are lightweight, and hence flexible. The structural flexibility results in significant elastic vibrations that are caused not only by exogenous perturbations but are also caused by maneuvers like slewing. Recent works have solved the output tracking problem, for example, given a desired output trajectory, inversion-based techniques find input-state trajectories that exactly track the output (Bayo, 1987; Kwon and Book, 1990; Ledesma et al., 1994; Devasia et al., 1996; Devasia, 1997). These inversion-based techniques have been successfully applied to the control of multi-joint flexible manipulators in Moulin and Bayo (1991); Paden et al. (1993), and to aircraft control in Meyer et al. (1995); Martin et al. (1996); Tomlin et al. (1995).

If the number of actuators are the same as the number of tracked-outputs (square system), then the inverse is *unique*. For a desired output trajectory, the inverse technique finds the unique bounded input-state trajectory, that can achieve exact-tracking. Although such an input-state trajectory exactly tracks the desired output, it might not meet other performance requirements in flexible structures. For example, during slewing maneuvers of a flexible manipulator, the structural deformations—determined by the inverse state trajectories—may be unacceptable. Further, excessively large actuator-inputs might be needed during an exact tracking maneuver (Tomlin et al., 1995). If the number of actuators are more than the number of outputs to be tracked, then the actuator redundancy can be used to optimally minimize the actuator-inputs and to reduce structural vibrations (Devasia and Bayo, 1994). However, if such actuator redundancy is not available then a compromise is desired between the tracking requirement and other goals like the reduction of internal vibrations and prevention of actuator saturation—the output trajectory needs to be redesigned.

The problem of redesigning an input to the system to minimize residual and in-maneuver vibrations for flexible structures has been well-studied in literature—see for example Singer and Seering (1990), Aspinwall (1980), Swigert (1980), Farrenkopf (1979), Bhat and Miu (1990), Smith (1958). In tracking problems, however, outputs are usually specified and not the inputs—input trajectories have to be computed from the desired outputs. For nonminimum phase systems (e.g., flexible structures with non-collocated sensors and actuators), the inputs to the system are difficult to determine and require the inversion of the system dynamics. Thus, if an output-maneuver is being designed then typical input-modification based approaches are not *directly applicable*. Rather than (a) find the input through inversion and then (b) optimize the inverse input, this paper describes a method to directly solve the optimal-inverse problem.

One existing approach to solve the output-redesign problem is to extend the input-redesign problem to the output-redesign problem. Such an approach: (a) chooses a feedback-based tracking controller—the desired output trajectory is now an input to the closed-loop system; and (b) redesigns this *input* to the closed-loop system. Thus, the output is redesigned (Singer and Seering, 1990). These redesigns are, however, dependent on the choice of the tracking controllers (Cook, 1993) because the controller optimization and trajectory redesign problems become coupled—this coupled optimization is still an open problem. An additional problem with this approach is that purely feedback-based output tracking may not yield satisfactory tracking due to performance-limitations of feedback-based regulators for nonminimum phase systems (Qui and Davison, 1993).

In contrast to input-redesign, we present an approach to directly redesign a given output-maneuver y_d by developing an optimal-inversion approach. This approach also finds a feedforward input trajectory that achieves exact tracking of the modified output-maneuver. Any errors in the tracking the modified output trajectory, y_{opt} (due to, for example, initial conditions and modeling errors) can then be be corrected using standard feedback approaches, i.e., stabilize the state trajectory, x_{opt}, which exactly tracks the optimal output, y_{opt} (see, for example, Khalil, 1991). For example, the feedback-control that stabilizes the state trajectory can be chosen as Ke, where e is the tracking error and K is the feedback controller gain. During the exact-inversion-based trajectory redesign, the feedback controller is inactive because the tracking error, e, is zero, and therefore the trajectory redesign is independent of the particular choice of the feedback law (i.e., the choice of K). In this sense, the optimal-inversion-based output-redesign problem is decoupled from the choice of a particular feedback controller needed to stabilize the system. This approach is illustrated with an example in this paper.

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We pose the output trajectory redesign problem as an optimization of a general quadratic cost function as in Chun et al. (1985), and solve it in the context of linear systems. The formulation allows for the minimization of both: (*a*) residual vibrations and (*b*) vibrations during the maneuver. Such in-maneuver vibration-reduction is required for tracking maneuvers of flexible spacecraft (see, for example, Chun et al., 1985, and recent command-shaping techniques in Singhose et al., 1996, 1997). The redesigned output trajectory is obtained by passing the initial output-trajectory through a pre-filter and can be implemented using a convolution similar to the convolutions prevalent in command shaping approaches (see for example Singer and Seering, 1988, 1990).

The criterion for the proposed output-trajectory redesign can be defined in terms of a quadratic cost functional; this cost criterion can be chosen to obtain a trade-off between the precision in tracking, and the reduction of structural vibrations and inputs. For a particular cost criterion, the optimal input can be obtained using a prefilter. An advantage of the present technique is that this prefilter only depends on the choice of the optimization criterion (choice of weighting matrices in a quadratic cost functional) and does not depend on the particular output trajectory. Thus, for a given optimization criterion, the prefilter can be precomputed independent of the output.

If the redesign of a particular output-trajectory is desired, then it is also possible to use the proposed approach for satisfying other criterion—like the prevention of actuator saturation. This can be achieved by manipulating the weighting matrices in the cost functional as in standard linear quadratic optimal control approaches. However, in such redesigns, the appropriate choice of the cost-criterion will depend on the particular output to be tracked. Thus, the resulting output-redesign will also depend on the particular output trajectory.

We begin by formulating the optimal output-trajectory redesign problem, and then solve it in the context of general linear systems. This theory is then applied to an example flexible structure and experimental results are provided.

2 Problem Formulation and Solution

System Inversion for Exact Tracking. Let the system dynamics be described by

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where $x \in \Re^n$, $u \in \Re^p$ and $y \in \Re^p$. The inversion approach (see, for example, Devasia et al., 1996) finds a bounded input-state trajectory that satisfies the above system equations, and yields the exact desired output, i.e.,

$$\dot{x}_{ref} = Ax_{ref} + Bu_{ff}$$
$$y_d = Cx_{ref}.$$

The inverse input-state trajectories can be described in terms of Fourier transforms as (Bayo, 1987)

$$\begin{aligned} u_{ff}(j\omega) &= [C(j\omega I - A)^{-1}B]^{-1}y_d(j\omega) \\ &:= G_y^{-1}(j\omega)y_d(j\omega) \\ x_{ref}(j\omega) &= [(j\omega I - A)^{-1}B]u_{ff}(j\omega) \\ &:= G_x(j\omega)u_{ff}(j\omega). \end{aligned} \tag{1}$$

This Fourier-based inversion approach has been extended to nonlinear, time-varying, nonminimum-phase systems in Devasia and Paden (1998). However, we restrict our present discussion to linear time-invariant systems.

Remark 1. We note two results. One, an inverse exists if the output and a certain number of its time-derivatives are bounded. The number of time derivatives of the output, that needs to be specified for an inverse to exist, is well-defined and depends on the relative degree of the system (Isidori (1989); Devasia et al. (1996)). Second, for linear systems, with the same number of inputs as outputs, if the inverse exists then it is unique (Bayo, 1987; Kwon and Book, 1990; Ledesma et al., 1994; Devasia et al., 1996).

The Performance Criterion. Trajectory redesign seeks a compromise between the goal of tracking the desired trajectory and other goals like reducing inputs and vibrations. We formulate this redesign problem as the minimization of a quadratic cost function of the type

$$\int_{-\infty}^{\infty} \{u(t)^T Ru(t) + x(t)^T Q_x x(t) + [y(t) - y_d(t)]^T Q_y [y(t) - y_d(t)]\} dt \tag{2}$$

where R, Q_x, and Q_y are symmetric matrices that represent the weights on control-input, states, and the output-tracking-error, respectively. y_d is a desired output-trajectory which has an inverse (for requirements on y_d, see Bayo, 1987; Devasia et al., 1996). Using Parseval's theorem we rewrite our optimization problem in the frequency domain as the minimization of the cost functional

$$J := \int_{-\infty}^{\infty} \{u(j\omega)^* Ru(j\omega) + x(j\omega)^* Q_x x(j\omega) + [y(j\omega) - y_d(j\omega)]^* Q_y [y(j\omega) - y_d(j\omega)]\} d\omega \tag{3}$$

where the superscript * denotes complex conjugate transpose.

Optimal Redesign of the Output. Our main result is given by the following lemma, which shows that the optimal output-trajectory redesign can be described as a prefilter, which maps a given desired output trajectory, y_d, to its redesigned output trajectory, y_{opt}. For a given cost functional, the pre-filter, G_f, doesn't depend on the particular choice of desired trajectory. Thus, the prefilter can be pre-computed.[2]

Lemma

- The modified output trajectory, y_{opt}, is given by
$$y_{opt}(j\omega) = G_f(j\omega)y_d(j\omega)$$
where
$$G_f(j\omega) = 1 - G_y[R + G_x^* Q_x G_x + G_y^* Q_y G_y]^{-1}[R + G_x^* Q_x G_x]G_y^{-1}.$$

- The modified input trajectory u_{opt}, that exactly-tracks the optimal output y_{opt}, is given by
$$u_{opt}(j\omega) = u_{ff}(j\omega) + v_{opt}(j\omega)$$
where
$$v_{opt}(j\omega) = G_v(j\omega)y_d(j\omega)$$
and
$$G_v(j\omega) = -(R + G_x^* Q_x G_x + G_y^* Q_y G_y)^{-1}(R + G_x^* Q_x G_x)G_y^{-1} \tag{4}$$

- The modified state trajectory x_{opt}, which is found through inversion of the optimal output y_{opt}, is given by
$$x_{opt}(j\omega) = G_x(j\omega)G_y^{-1}(j\omega)G_f(j\omega)y_d(j\omega)$$

[2] MATLAB code for optimal inversion of single-input single-output systems can be obtained by email to santosh@eng.utah.edu.

Proof: Without loss of generality we rewrite the input u as the sum of the feedforward input, $G_y^{-1}y_d$, found from inversion of the desired trajectory, and an arbitrary v

$$u(j\omega) = u_{ff}(j\omega) + v(j\omega)$$
$$= G_y^{-1}(j\omega)y_d(j\omega) + v(j\omega). \quad (5)$$

Substituting $x(j\omega) = G_x(j\omega)u(j\omega)$, and $y(j\omega) = G_y(j\omega)u(j\omega)$ along with the input u found from Eq. (5) into the cost function given by Eq. (3), we obtain

$$J = \int_{-\infty}^{\infty} \{ [v + (R + G_x^* Q_x G_x$$
$$+ G_y^* Q_y G_y)^{-1}(R + G_x^* Q_x G_x)G_y^{-1}y_d]^* \times (R + G_x^* Q_x G_x$$
$$+ G_y^* Q_y G_y) \times [v + (R + G_x^* Q_x G_x + G_y^* Q_y G_y)^{-1}$$
$$\times (R + G_x^* Q_x G_x)G_y^{-1}y_d] + (G_y^{-1}y_d)^*[(R + G_x^* Q_x G_x)$$
$$- (R + G_x^* Q_x G_x)^* \times (R + G_x^* Q_x G_x$$
$$+ G_y^* Q_y G_y)^{-1}(R + G_x^* Q_x G_x)](G_y^{-1}y_d)\}dw$$

Note that the first term (enclosed in square brackets) in the cost function is quadratic and the cost function can be minimized by setting this quadratic term to zero, i.e., by choosing

$$v(j\omega) = v_{opt}(j\omega) = G_v(j\omega)y_d(j\omega),$$

where G_v is defined by Eq. (4) in the Lemma. The choice of $v = v_{opt}$ defines the optimal input u_{opt} through Eq. (5) as

$$u_{opt}(j\omega) = [G_y^{-1}(j\omega) + G_v(j\omega)]y_d(j\omega). \quad (6)$$

The result follows from

$$y_{opt}(j\omega) = G_y(j\omega)u_{opt}(j\omega)$$
$$= [1 + G_y(j\omega)G_v(j\omega)]y_d(j\omega). \quad \square$$

Remark 2. We point out two extreme cases. First case: if the weight on the tracking error is zero, $Q_y = 0$, but R is positive definite then we obtain $v = -(G_y^{-1}y_d) = -u_{ff}$. This implies that the input $u_{opt} = u_{ff} + v = 0$, i.e., the best strategy is not to track the desired trajectory at all. Second case: if the weight on the inputs and states are zero, i.e., $R = 0$ and $Q_x = 0$ but with Q_y positive definite then $y_{opt} = y_d$. This implies that exact output-tracking is optimal, and the cost is again zero.

Remark 3. If the cost function is defined in the frequency domain (as in equation (3)) then the weighting matrices R, Q_x and Q_y can be frequency dependent—such frequency dependent weights can be used, for example, to account for actuator-bandwidth-limitations (Gupta, 1980).

3 Example

Experimental System. Consider an experimental flexible structure which consists of two discs connected by a thin shaft as shown in Fig. 1. These two discs can rotate freely. The

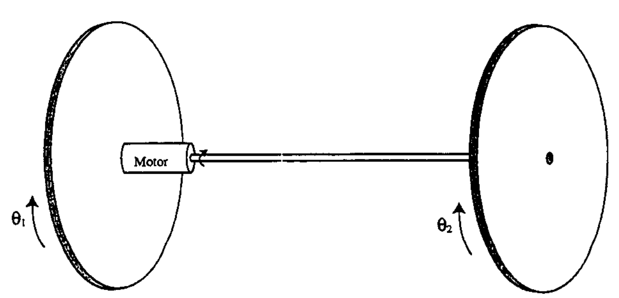

Fig. 1 Schematic experimental setup

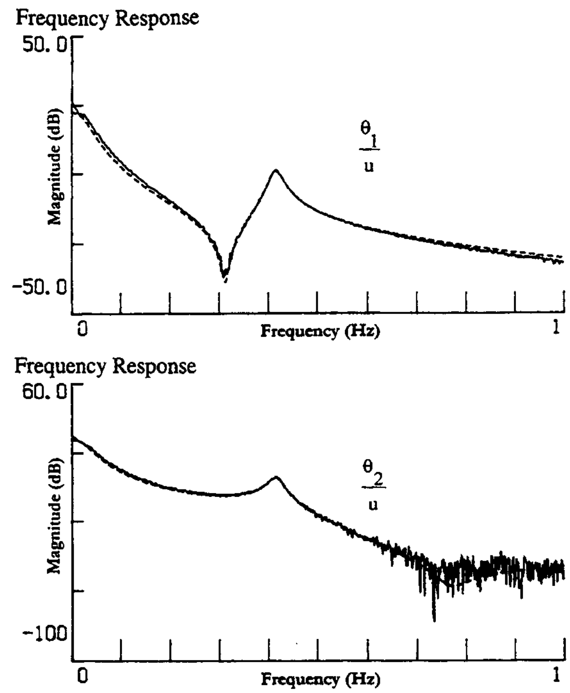

Fig. 2 Bode plots. (——) (- - -) model

transfer function of the system, approximated by a rigid body mode and one flexible mode, was obtained experimentally using a HP3562A Dynamic Signal Analyzer. Input, u, to the system is the voltage (volts) applied to a DC motor, and the outputs are the angular rotations (in degrees) of the two disks θ_1, θ_2. These angular rotations are measured using potentiometers, and the transfer functions are obtained as

$$\frac{\theta_1}{u} = \frac{21.903S^2 + 1.3465S + 84.2}{S^4 + .512S^3 + 6.937S^2 + 2.444S + 0.3186}$$
$$\frac{\theta_2}{u} = \frac{3.588S^2 - 1.186S + 84.2}{S^4 + .512S^3 + 6.937S^2 + 2.444S + 0.3186}. \quad (7)$$

A comparison, of the frequency response plots of the above transfer functions with the experimental plots (see Fig. 2), shows that the one-flexible-mode approximation is sufficient to adequately model the plant. The large inertia at the two ends of the flexible-shaft (Fig. 1) creates a very low fundamental vibrational frequency (around 0.4 Hz)—the structural vibrations also have a low damping coefficient. It is noted that the structure tends to settle in one particular equilibrium configuration due to a small imbalance in the mass distribution (i.e., the structure has a static imbalance) which is not modelled by these transfer functions. Further, this linear model also has errors in modeling the friction at the bearings.

With the state vector, x, chosen as $x := [\theta_1\ \dot{\theta}_1\ \theta_2\ \dot{\theta}_2]^T$ the system equations can be represented in state-space form as

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3.6562 & -0.4359 & 3.5734 & -0.0910 \\ 0 & 0 & 0 & 1 \\ 3.2453 & -0.1256 & -3.2589 & -0.0764 \end{bmatrix} x + \begin{bmatrix} 0 \\ 21.903 \\ 0 \\ 3.588 \end{bmatrix} u$$
$$:= Ax + Bu$$
$$y := \theta_2 = [0\ 0\ 1\ 0]x. \quad (8)$$

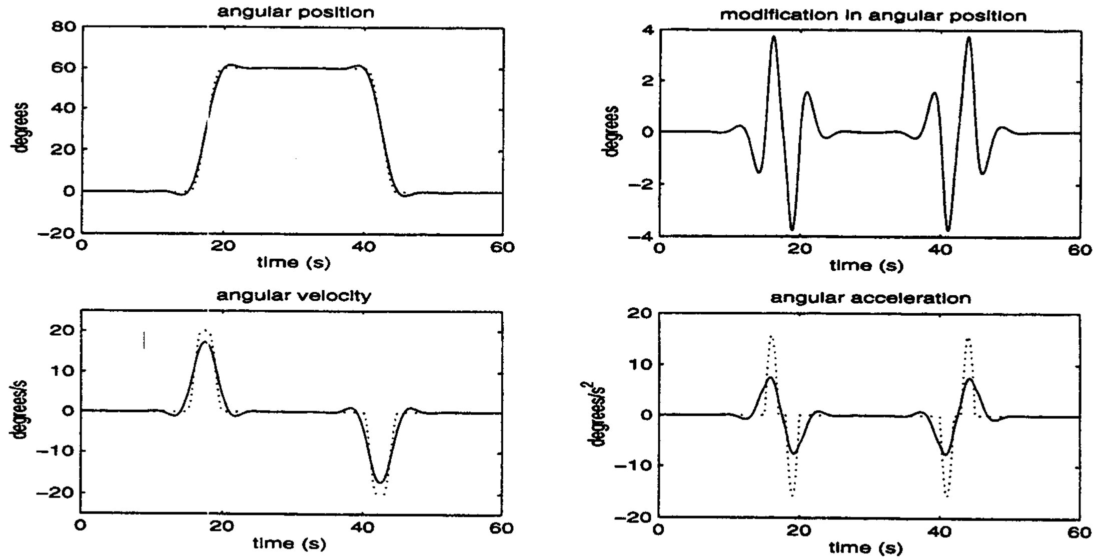

Fig. 3 Desired and modified output profiles. Dotted line represents desired values; solid line represents modified values.

Output Redesign. The control objective is to track the angular rotation, θ_2 of the disk, that is farthest away from the motor (see Fig. 1). The desired output trajectory used in the example, and its time-derivatives are shown in Fig. 3. For this particular example, the second derivative of the desired output, i.e., the desired angular acceleration profile of the output, uniquely determines the inverse, exact-tracking, input-state trajectory (since the relative degree is two (Isidori, 1989). Thus, the acceleration, specified by the desired output, also determines the resulting structural vibration, $\theta_1 - \theta_2$. If the internal vibrations are to be reduced, then we have to relax the exact-tracking requirement. Similarly, to reduce the required input amplitudes, we have to compromise exact-tracking. This trade-off can be represented as the minimization of a general quadratic cost function given in Eq. (2) with $R = r$, $Q_y = q_y$, and

$$Q_x = q_x \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The scalars r, q_x, q_y represent the relative weights on the reduction of inputs, structural vibrations and output-tracking errors, respectively. For example, increasing r tends to reduce the inputs used. In the simulations and experiments, these constants were chosen as $r = 1$, $q_x = 5$, and $q_y = 1$. Note that for this particular choice of the weights, the prefilter is independent of the particular output trajectory to be redesigned.

Remark 4. For a particular output trajectory, the weights, r, q_x, and q_y can be manipulated to achieve goals, like the prevention of actuator saturation. However, in such redesigns the appropriate choice of the cost-criterion (and the resulting prefilter) will depend on the particular trajectory.

Results. The output was redesigned and the performance improvement was evaluated in terms of elastic deflections, $\theta_1 - \theta_2$, of the flexible shaft and in terms of the inputs to the actuator, u. The modification of the desired trajectory is shown in Fig. 3. The input-state trajectory was found through inversion as described in Eq. (1). Two sets of inverses were found: first with the original trajectory, y_d, and second with the modified output trajectory, y_{opt}. Figure 4 compares the original and modified input-state trajectories found from inversion. The state tra-

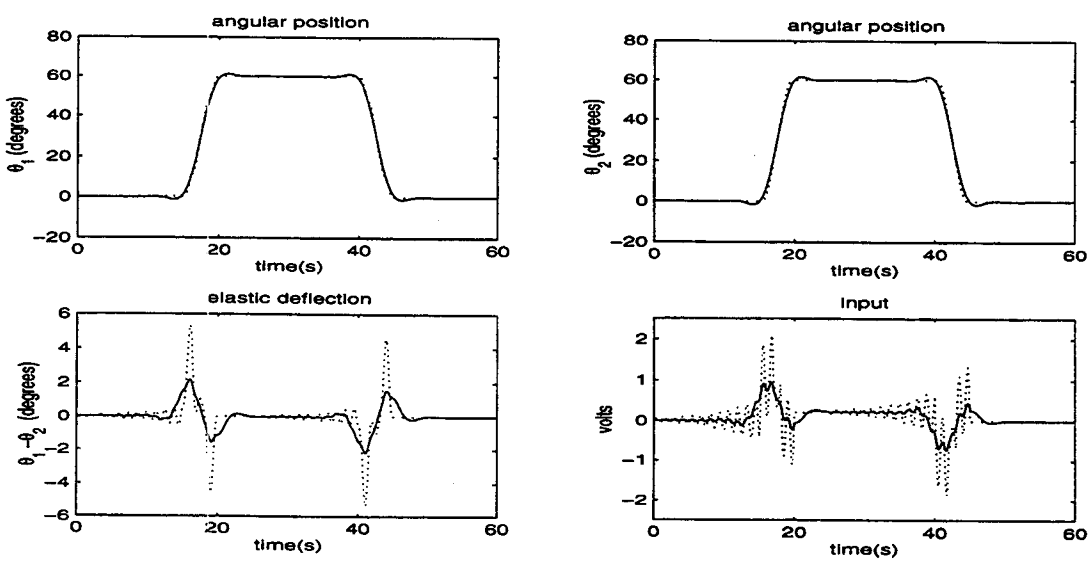

Fig. 4 Inverse of desired and modified output profiles. Dotted line represents desired values; solid line represents modified values.

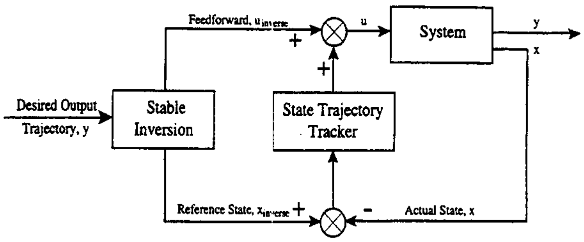

Fig. 5 The control scheme

jectories, found from inversion, were stabilized through feedback (see control scheme in Fig. 5)—note that the trajectory redesign is independent of the choice of the feedback law. Experimental results are presented in Fig. 6.

The experimental results shown in Fig. 6 confirm that relatively small modifications of the output trajectory (Figs. 3 and 6) can lead to substantial reductions in inputs and elastic vibrations. From the experimental data (Fig. 6), the root mean square (rms) value of the output-modification was 2.41 degrees for a maneuver that has a maximum desired slew of 60 degrees. With this modification of the output trajectory, the rms value of the input was decreased by 47.5 percent, and rms value of the twist in the flexible shaft was decreased by 34.6 percent.

4 Discussion

Recently developed inversion-based approaches achieve output tracking by finding feedforward inputs and reference state trajectories that yields exact output-tracking. Such an inverse input-state trajectory is unique for a given output trajectory. Therefore, the accompanying vibrations and inputs (resulting from applying the inversion-based controller) may not be acceptable. Figures 3 and 4 show that output-tracking can be traded-off (i.e., the output can be modified) to achieve substantially lower vibrations and inputs.

It is noted that an output modification does not necessarily imply that a substantial increase in maneuver time is needed. However, if a substantial decrease in vibration is required (i.e., a large q_x is chosen in the cost-functional) then the maneuver time can increase. However, if an increase in maneuver time is needed, then this increase is not a drawback of the approach, but is the result of the designer's choice of the cost-criterion, and the particular original output trajectory. Even for situations in which the trajectory redesign results in an increase in the time during which the output is changing, the total maneuver time (which should also include the time during which the input is non-zero) may not change significantly with output redesign. In the present example, the amount of preactuation time (during which the input is nonzero) is similar for, both, the original output trajectory and the modified output (compare input trajectories in Figs. 4 and 6). Thus, the effective maneuver time has not changed for the modified output trajectory.

The output-trajectory modification can include combinations of (a) increase in maneuver time and (b) smoothing of the output trajectory. Although it is intuitive that such modifications can lead to a decrease in magnitudes of vibrations and inputs, typical approaches are ad-hoc and do not exploit the knowledge of the system dynamics. In contrast, the present approach provides a more systematic approach, that can be used by the designer to achieve a trade-off between exact tracking and other requirements like vibration and input reductions.

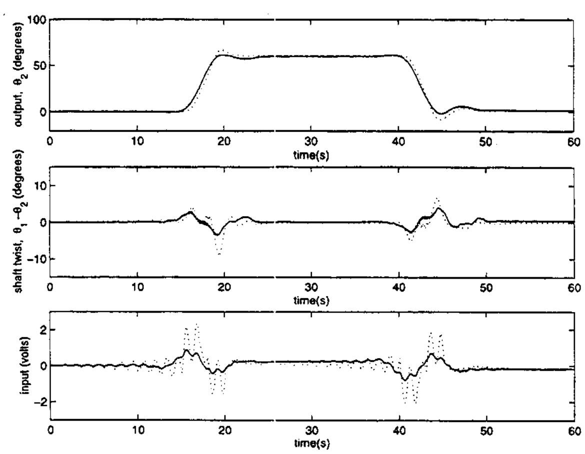

Fig. 6 Output redesign: experimental results. Dotted line represents tracking for the original trajectory and solid line represents tracking for the modified trajectory.

5 Conclusion

The trajectory redesign problem was formulated and solved in the context of linear invertible systems—including nonminimum phase systems. The approach provides a systematic approach to an optimal trade-off between tracking desired trajectory and other goals like vibration reduction and reduction of required inputs. The approach was applied to an example flexible structure, and experimentally verified. Future work will address trajectory redesign for nonlinear systems.

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