Application of Feedforward Dynamics Compensation in Ionic-Polymer Metal Composite Actuators

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ABSTRACT

Ionic-polymer metal composites are innovative materials that offer combined sensing and actuating ability in lightweight and flexible package. As such, they have been exploited in robotics and a wide variety of biomedical devices, for example, as fins for propelling aquatic robots and as an injector for drug delivery. One of the main challenges of IPMC-based actuators is precision control of their movements, especially at high operating speed (frequency) because of dynamic effects. As the frequency increases, the dynamics cause vibration which leads to significant tracking error. A model-based feedforward controller is applied to control the position of a custom-made Nafion-based IPMC actuator. The feedforward controller was designed to account for the linear dynamics, and the feedforward input was computed by considering the magnitude of the input signal and the tracking precision. To account for unmodeled effects not captured by the linear model, a feedback controller was integrated with the feedforward controller. The feedback controller provides robustness. Experimental results show a significant improvement in the tracking performance using feedforward control. In particular, the feedforward controller resulted in over 75% improvement in the tracking error compared to the case without dynamic compensation. Then by adding a proportional-integral feedback controller, the tracking error was less than 10% at 18 Hz scan frequency.

Keywords: Ionic polymer metal composite actuator, dynamic effects, feedforward control.

1. INTRODUCTION

This paper focuses on feedforward control to compensate for dynamic effects in Nafion-based ionic-polymer metal composite (IPMC) actuators. At high frequency, induced structural vibrations in the IPMC actuator leads to significant positioning error. For example, when an IPMC actuator is used to position an optical fiber for endoscopy, movement induced vibrations can cause image distortion. The distortion is a result of the discrepancy between the fiber’s actual and desired location at the time information is acquired. The induced vibrations are caused by exciting the resonant modes of the IPMC actuator. In other applications such IPMC-based mechanical grippers, robotic arms and fingers, and artificial ventricular muscles, movement-induced vibrations can severely limit performance. The contribution of this paper is the application of a frequency-weighted, inversion feedforward controller for high speed positioning of an IPMC actuator. The feedforward controller is designed to compensate for the dynamics which are significant at high frequency. To account for unmodeled dynamics, such as nonlinearities in the IPMC, a feedback controller was combined with the feedforward controller. The integrated controller (1) provides robustness and (2) improves the performance of the feedforward controller. Experimental results are presented to demonstrate the effectiveness of the control schemes.

Ionic-polymer metal composites are in the electroactive polymer family. Typical IPMCs consist of a Nafion ionic-polymer membrane sandwiched between inert metal electrodes. The metal electrodes are often platinum or gold. When an electric field is applied to this electrodes, the IPMC bends toward the anode. When an IPMC is strained, it generates a measurable voltage and thus it can be used as a sensor. Some advantages of IPMC actuators include: low drive voltage, easily shaped, silent motion, high displacement to mass ratio, and the ability to work in an aqueous environment. However, the behavior of IPMCs include nonlinearities, relaxation, dynamic effects, and time-varying behavior. Therefore, precision positioning of IPMCs is a challenge.

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Controllers for IPMCs fall into two general categories: (1) model-based feedforward control and (2) feedback control. The Preisach hysteresis model was applied to model the quasi-static hysteresis behavior in IPMC, and then the model was used to design a feedforward controller to compensate for the hysteresis effect.\(^5\) Feedback controllers have been studied for IPMC, for example, the traditional PID controller,\(^6,7\) an LQR state-space feedback controller,\(^8,9\) an adaptive controller\(^10\) and a robust controller.\(^11\) Although feedback control is relatively straightforward to implement, it is often bandwidth-limited.\(^12\)

The approach taken in this paper to control the movement of an IPMC actuator was to exploit a linear dynamics model of the actuator for feedforward control. First, the frequency response was experimentally measured using a dynamic signal analyzer. The frequency response captures the linear dynamics of the actuator, and then the measured response was used directly in an inversion-based feedforward approach\(^13\) to compute the input which accounts for the dynamics. One advantage of the inversion method is it is applicable to nonminimum phase systems.\(^14,15\) To account for unmodeled effects and to provide robustness, a feedback controller was designed and integrated with the feedforward controller. Experimental results show a significant improvement in tracking performance when the inverse feedforward input was applied.

This paper is organized as follows. Section 2 describes the feedforward controller design. Afterwards, the experimental system is described in Section 3.3. The implementation of the feedforward controller is presented in Section 4. Finally, experimental results and concluding remarks are found in Sections 5 and 6, respectively.

### 2. FREQUENCY-WEIGHTED INVERSION-BASED FEEDFORWARD CONTROL

A model-based, inversion feedforward approach\(^13,16\) was applied to control the IPMC actuator. Model-based feedforward control exploits known information about the system to compensate for effects such as dynamics\(^17\) and/or hysteresis.\(^5\) Thus, for a given desired output trajectory \(y_d\), the feedforward method finds the input \(u_{ff}\) that compensates or anticipates for deficit performance. The block diagram of the feedforward control scheme is shown in Fig. 1(a).

![Figure 1. The block diagrams of the control systems. (a) Model-based, inversion feedforward controller. (b) Integrated feedforward and feedback controller.](image)

This paper is concerned with accounting for the induced structural dynamics (vibrations) of the IPMC actuator at high operating speed. Let \(G(j\omega) = y(j\omega)/u(j\omega)\) represent the transfer function of the IPMC’s vibrational dynamics in the frequency domain. The applied voltage is considered the input \(u(j\omega)\) and the displacement of the IPMC actuator is considered the output \(y(j\omega)\). When the desired output trajectory \(y_d(j\omega)\) is known \textit{a priori}, the feedforward control input \(u_{ff}(j\omega)\) can be obtained directly,\(^14\)

\[
u_{ff}(j\omega) = G^{-1}(j\omega) y_d(j\omega).
\]  \(1\)

The time domain solution of the inverse input \(u_{ff}(t)\) is found by the inverse Fourier transform. Although the implementation of (1) is straightforward, the resulting inverse feedforward input may yield excessively large
inputs when the system has lightly-damped system zeros. The excessively large inputs (i.e., voltage) can damage the IPMC actuator. Additionally, large model uncertainties around the resonant peaks and/or lightly-damped zeros may cause significant error in computing the feedforward input. As a result, the inversion technique (1) may lack robustness.

The issues of modeling errors and dynamics variation can be addressed by the optimal inversion technique. Specifically, an optimal feedforward input is obtained by minimizing the quadratic cost function,

\[
J(u) = \int_{-\infty}^{\infty} \left[ u^*(j\omega)R(j\omega)u(j\omega) + e^*(j\omega)Q(j\omega)e(j\omega) \right] d\omega,
\]

where \( u \) denotes the input, \( e = y - y_d \) is the tracking error, and \( R(j\omega) \) and \( Q(j\omega) \) are non-negative, frequency-dependent real-value weights on the input energy and the tracking error, respectively. When the cost function (2) is minimized, the optimal feedforward input is

\[
u_{opt}(j\omega) = \left[ \frac{G^*(j\omega)Q(j\omega)}{R(j\omega) + G^*(j\omega)Q(j\omega)G(j\omega)} \right] y_d(j\omega).
\]

When the optimal inverse input \( u_{opt}(j\omega) \) is applied to the IPMC actuator represented by \( G(j\omega) \), the following modified output is tracked,

\[
y_{opt}(j\omega) = G(j\omega)u_{opt}(j\omega),
\]

\[
y_{opt}(j\omega) = G(j\omega) \left[ \frac{G^*(j\omega)Q(j\omega)}{R(j\omega) + G^*(j\omega)Q(j\omega)G(j\omega)} \right] y_d(j\omega),
\]

\[
\triangleq G_f(j\omega)y_d(j\omega).
\]

Therefore, \( G_f(j\omega) \) is a filter that modifies the desired trajectory \( y_d(j\omega) \) based on the \( R(j\omega) \) and \( Q(j\omega) \) weightings.

The weights \( R(j\omega) \) and \( Q(j\omega) \) can be designed to take into account the input magnitude and the model uncertainties over certain frequency ranges. Generally speaking, the input energy weight \( R(j\omega) \) is chosen much larger than the tracking error weight \( Q(j\omega) \) at those frequencies (a) where there are large model uncertainties and/or (b) around lightly-damped zeros.

In addition to the vibrational dynamics, the behavior of IPMCs may include hysteresis and relaxation. These additional effects were not considered in this work. Particularly, the electro-mechanical model \( G(j\omega) \) is found over a relatively-high frequency range. By doing this, the relaxation behavior that occurs over long periods of time was ignored. Likewise, the effect of hysteresis was not considered. The model \( G(j\omega) \) was obtained over the IPMC actuator’s linear range, i.e., by assuming that over relatively small range motion hysteresis effect was negligible.

To account for these unmodeled effects, the optimal inverse input \( u_{opt}(t) \) and the modified output \( y_{opt}(t) \) were integrated with a feedback controller. The block diagram of the integrated control system is shown in Fig. 1(b). The feedback controller (1) provides robustness to variations in the dynamics and (2) accounts for the hysteresis and relaxation effects. The modified trajectory \( y_{opt}(t) \) became the reference trajectory to the feedback system, i.e., \( y_{ref}(t) = y_{opt}(t) \).

3. THE EXPERIMENTAL IPMC ACTUATOR SYSTEM

The IPMC actuators used in all experiments were fabricated by following established recipes. The fabrication process and the experimental system are briefly described below.

3.1. Fabricating Nafion IPMC Actuators via the Casting Method

Nafion polymer membrane for making IPMC actuators is commercially available from DuPont. The other option is to make the membrane from Nafion solution. Membranes can be made to a desired thickness. To create the membrane, liquid Nafion solution (5% Nafion by weight) was poured into a Pyrex glass mold as shown in Fig. 2. Then the solution was dried in air for approximately two days. Afterwards, the membrane was released from the mold and trimmed as shown. The membrane went through heat treatment in an oven at 140°C under light pressure \((1.24 \times 10^4 \text{ Pa})\) for 30 min. The average thickness of the fabricated membrane was 0.38 mm.
3.2. Depositing Platinum Electrodes

The next step was to electro-chemically deposit the platinum electrodes. Platinum was preferred because of the acidic nature of the ionic membrane. The metal does not oxidize when voltage is applied — oxidation corrodes the metal and reduces the conductivity of the electrode. The ingredients of the electro-chemical process include: the Nafion membrane, platinum complex ($Pt(NH_3)_4Cl_2$), 1 molar sulfuric acid ($H_2SO_4$), deionized water, and 1% (wt/wt) sodium borohydride ($NaBH_4$).

First, the Nafion membranes were cleaned in an ultrasonic bath with deionized water. Then, the pieces were boiled in 1 molar sulfuric acid ($H_2SO_4$) for 30 minutes. Afterwards, they were removed from the acid solution and boiled in deionized water for another 30 minutes.

A tetramine platinum chloride ($Pt(NH_3)_4Cl_2$) solution was created by mixing 720 mg of tetramine platinum chloride with 360 ml of deionized water. The solution met the minimum requirement of 3 mg of platinum for every cm$^2$ of surface area of the Nafion membrane, and no less than 2 mg of platinum for every ml of water. The polymer membranes were then soaked in the solution followed by occasional stirring for approximately 16 hours at room temperature.

Next, a 1% (wt/wt) solution was created by mixing 280 ml of deionized water with 2.8 g of sodium borohydride ($NaBH_4$). The membranes were placed vertically in test tubes with deionized water at 40°C. Then 4 ml of sodium borohydride solution per 60 cm$^2$ of Nafion membrane sample was added every 30 minutes for 3 hours. The quantity of the reagent was kept in proportion to the area of the sample. Next, 40 ml of the solution was added for each 60 cm$^2$ sample and the temperature was gently raised to 60°C. Two hours later, the samples were removed from the test tubes and washed in deionized water. This completed the process to deposit one layer of platinum on the Nafion membrane surface.

More layers were created by repeating the process. Five layers of platinum were deposited to create the experimental IPMC actuators.
3.3. The Experimental System

The fabricated IPMC membrane was cut into 7 mm × 24 mm strips and used as actuators. Each actuator was clamped at one end by a custom-designed stainless steel fixture as depicted in Figs. 3(a) and (b). The other end of the actuator was free to move (cantilever configuration). A desktop computer with a 12-bit digital-to-analog converter board supplied a reference input voltage $u$ to a custom-designed voltage amplifier circuit. The output of the circuit was used to drive the IPMC actuator. The IPMC’s displacement ($y$) was measured with a noncontact reflective laser sensor (SUNX micro laser sensor LM10, model No. ANR12511 and controller model No. ANR5232). The sensor’s resolution was 1 µm, and it was positioned at approximately $d = 55$ mm from the IPMC actuator as shown in the Fig. 3. The sensor signal was collected with a 12-bit analog-to-digital converter board and processed by the desktop computer. The computer was used to implement the controllers and it has a closed-loop bandwidth of 20 kHz.

![Diagram of the experimental setup](image)

**Figure 3.** The experimental IPMC system. (a) and (b) Photographs of the IPMC actuator mounted in cantilever configuration. The displacement of the actuator ($y$) is measured by a laser sensor (SUNX micro laser sensor LM10, model No. ANR12511 with sensor controller, model No. ANR5232). (c) The block diagram of the experiment.

3.4. Open-Loop Step Response of the IPMC Actuator

Several experiments were performed to assess the open-loop response the IPMC actuator before application of feedforward and feedback control. The open-loop step response of an IPMC actuator is shown Fig. 4. Three input voltages were applied to the IPMC actuator: 0.5 V, 1.0 V, and 2.5 V. The corresponding measured displacements (outputs), normalized with respect to the peak value ($\pm 2.0$ mm), are shown in Fig. 4(b). In the figure, it is apparent that upon application of a step voltage, the IPMC initially responds with a quick change in displacement toward a maximum value. The 5% settling time was approximately $t_s = 170$ s. Upon reaching the maximum value, the IPMC slowly relaxed. This behavior has been observed by other researchers.1,22,23
This work was concerned with the high-frequency behavior (vibrational dynamics) of the IPMC shown in the inset plot, Fig. 4(c). The relaxation effect was referred to as the low-frequency behavior and it was not considered. In all experiments the IPMC was operated at relatively high frequency ($\geq 1$ Hz) to avoid the effects of relaxation. Moreover, the motion of the actuator was centered about the origin.

![Figure 4](image_url)

**Figure 4.** IPMC open-loop step response. (a) The input signal (in volts) versus time. (b) The measured output displacement (normalized) versus time showing the high frequency transient and the low frequency relaxation behavior. (c) Inset figure shows more closely the high frequency transient. The 5% settling time of the transient is approximately 170 ms.

### 4. FEEDFORWARD CONTROL IMPLEMENTATION

The feedforward control method described above required the knowledge of $G(j\omega)$. A model can be obtained by curve fitting the measured frequency response of the IPMC actuator over an appropriate frequency range. For example, the Matlab system identification package can be used to identify the model $G(j\omega)$ from measured input-output data. Then the model can be inverted to find the feedforward input as described above.

Rather than curve fit the measured frequency response curves to find $G(j\omega)$, the measured frequency response data was used directly in the inversion feedforward scheme (3, 4). This approach eliminated the time-intensive step of identifying the model $G(j\omega)$. Furthermore, the modeling step can introduce modeling errors. Figure 5 outlines the simplified implementation process.

First, the frequency response $G(j\omega)$ of the IPMC actuator was measured using a dynamic signal analyzer (DSA, Hewlett Packard model 35670A). (Other methods to obtain the frequency response curve from measured input-output data can be used.) A sinusoidal input voltage $u$, with a fixed amplitude and varying frequency, was applied to drive the IPMC actuator. The magnitude of the input was maintained at 500 mV. The output of the sensor was fed back to the DSA to construct the frequency response plots (magnitude and phase versus frequency). The frequency range was 1 Hz to 1 kHz and is shown in Fig. 6. The frequency response shows a dominate resonant peak at approximately 113 Hz. Therefore, when the IPMC was operated at relatively high frequencies, the resonant peak can be excited causing unwanted oscillations in the output response. The objective was to compensate for these effects using the feedforward method.

The frequency response $G(j\omega)$ was used directly to determine the optimal feedforward input $u_{opt}(j\omega)$ from (3). The specified desired output trajectory $y_d(t)$ was transformed into the frequency domain using the fast Fourier transform algorithm in Matlab. The tracking error and input weights $Q(j\omega)$ and $R(j\omega)$, respectively, were chosen as follows: $Q = 1$ and $R = 0$ for $\omega \in [0, 130]$ Hz and $Q = 0$ and $R = 1$ for $\omega > 130$ Hz. The weights were chosen such that the computed feedforward input magnitude was less than 5 V. Application of optimal
inversion feedforward scheme (3, 4) produced the feedforward input $u_{opt}(j\omega)$ and reference trajectory $y_{opt}(j\omega)$. The time-domain solutions $u_{opt}(t)$ and $y_{opt}(t)$ were found by the inverse Fourier transform.

Finally, the feedforward input $u_{opt}(t)$ and reference output $y_{opt}(t)$ were applied to the IPMC and closed-loop control system shown in Fig. 1. The results are discussed next.

5. EXPERIMENTAL RESULTS AND DISCUSSION

The experimental results for tracking a 1, 10, and 18 Hz triangular trajectory with amplitude 2.0 mm are presented below.
5.1. Feedforward Tracking Results
The feedforward method was applied to track the desired triangle trajectories and the results are shown in Fig. 7. The plots in Fig. 7 show the normalized displacement of the IPMC actuator with respect to time. The desired triangle trajectory $y_d(t)$ is denoted by the dash line. The dash-dot line represents the response of the IPMC actuator without feedforward compensation, that is, by applying the input

$$u(t) = \frac{1}{G(0)} y_d(t),$$

(5)

where $G(0)$ is the dc gain of the IPMC system. In all three cases, plots (a) through (c), the tracking error of the uncompensated case was significant. Table 1 lists the magnitude of the maximum tracking error,

$$e_{\text{max}}(\%) = \left[ \frac{|\hat{y} - \hat{y}_d|}{\max(\hat{y}_d) - \min(\hat{y}_d)} \right] \times 100\%,$$

(6)

where $\hat{y} = y / \max(y)$ and $\hat{y}_d = y_d / \max(y_d)$ are the normalized measured and desired output, respectively. Also listed are the root-mean-squared error values, defined as

$$e_{\text{rms}}(\%) = \left[ \frac{1}{T} \int_0^T \left( \hat{y}(t) - \hat{y}_d(t) \right)^2 dt \right]^{1/2} \times 100\%.$$

(7)

The results show clearly that the feedforward controller was able to improve the tracking response of the IPMC by at least 70% compared to the uncompensated case. Although the improvement was significant, at the 18 Hz scan frequency, the maximum tracking error with feedforward control was quite evident at 17.54%. The large tracking errors at high frequency could have been attributed to dynamic effects that were not captured by the measured frequency response $G(j\omega)$. To further improve the response of the feedforward controller, a feedback controller was designed and integrated with the feedforward controller.

Table 1. Feedforward control results: maximum and root-mean-squared tracking error relative to range for without and with inverse feedforward (FF) control.

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<tr>
<th>Traj. freq. (Hz)</th>
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<th>$e_{\text{max}}$ (%) [w/ FF]</th>
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<td>55.08</td>
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5.2. Feedforward and Feedback Controller Tracking Results
The feedforward controller compensated for the dynamic effects, however, residual error remained due to unmodeled effects. To minimize the residual error, a proportional-integral (PI) feedback controller $C(s) = K_p + \frac{K_i}{s}$, was combined with the feedforward controller [see Fig. 1(b)]. The constants were tuned experimentally to $K_p = 24$ and $K_i = 0.7$. The PI feedback controller was implemented on the computer with a closed-loop bandwidth of 10 kHz. One of the benefits of feedback control was it provided robustness to parameter or dynamics variation. Therefore, the feedback controller can be used to account for external disturbances or unmodeled dynamics.

The performance of the feedback controller is shown in Fig. 8, and immediately the PI controller shows good tracking performance at low frequency (1 Hz). However, as the frequency increases, the PI feedback controller’s performance begins to degrade. Table 2 shows that maximum error at 18 Hz is over 33%.
Figure 7. Feedforward control results. (a) 1 Hz tracking. (b) 10 Hz tracking. (c) 18 Hz tracking.

Figure 8. Feedback controller results. (a) 1 Hz tracking. (b) 10 Hz tracking. (c) 18 Hz tracking.
Table 2. PI feedback control results: maximum and root-mean-squared tracking error relative to range for without and with PI feedback (FB) control.

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When the feedback controller was combined with the feedforward controller, the error was reduced considerably compared to both the uncompensated case shown in Fig. 7 and the feedback-only case shown in Fig. 8. At 18 Hz scan frequency, the maximum tracking error for the integrated controller was just over 7%. Table 3 lists the tracking results for integrated controller. Comparing the results of Table 1 and Table 3, the feedback controller reduced the maximum tracking error of the feedforward controller at 18 by over 50%. Therefore, the combination of feedforward and feedback control allows precise tracking control of IPMC at relatively high scan frequency.

![Graphs showing integrated feedforward and PI feedback controller results at 1 Hz, 10 Hz, and 18 Hz tracking frequencies.](attachment:image.png)

**Figure 9.** Integrated feedforward and PI feedback controller results. (a) 1 Hz tracking. (b) 10 Hz tracking. (c) 18 Hz tracking.
6. CONCLUSIONS

This paper presented the application of a frequency-weighted, inversion-based feedforward controller to compensate for the vibrational dynamics of an IPMC actuator. The controller was implemented by using the measured frequency response of the IPMC actuator, thus eliminating the process of identifying the dynamics model. Experimental results show that the feedforward controller compensated for the dynamics at relatively high scan frequency. The performance of the feedforward controller was improved by augmenting a PI feedback controller. The feedback controller reduced the tracking error of the feedforward controller by over 50% at 18 Hz.

Acknowledgements

Authors gratefully acknowledge the support of the National Science Foundation, Grant DUE #0633098. Additionally, the authors would like to acknowledge Robert Brik for his help with fabricating some of the IPMC actuators used in the experiments.

Table 3. Feedforward and feedback PI feedback control results: maximum and root-mean-squared tracking error relative to range for PI feedback only (FB only) control and integrated PI feedback and feedforward control (FF & FB).

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